3rd Week

8 Mart 2023 Çarşamba 14:33

Fixed quizzes available every Saturday 9 - 9
[20 min - (25 min)] 2-3 question

→ Random quizzes in-class 1-question }

1) $r_i \leftrightarrow r_j$
2) $cr_i \to r_i$ } → obtain a "leading 1"
3) $cr_i + r_j \to r_j$ → making zeros downside (/upside) of a leading 1. → REF / RREF

Ex
system → augmented matrix

$$\begin{array}{c} r_1 \\ r_2 \\ \\ \\ \\ \end{array}\begin{bmatrix} 1 & -3 & \# \\ 0 & 4 & \\ 0 & 3 & \\ 0 & 5 & \\ 0 & 6 & \end{bmatrix} \xrightarrow{\frac{1}{4}r_2 \to r_2} \begin{bmatrix} 1 & -3 & \# \\ 0 & 1 & \# \quad \# \quad \# \\ 0 & 3 & \\ 0 & 5 & \\ 0 & 6 & \end{bmatrix}$$

make these zeroes

$r_1 + r_3 \to r_3$

$-15/8$, $93/12$

$$\xrightarrow{\substack{-3r_2 + r_3 \to r_3 \\ -5r_2 + r_4 \to r_4 \\ -6r_2 + r_5 \to r_5}} \begin{bmatrix} 1 & -3 & \\ 0 & 1 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \end{bmatrix}$$

REF

System of LE — Matrices

Matrix Algebra

(world) space of matrices → $\mathbb{R}^{m\times n}$ — m×n matrices (m rows, n column)

entries of matrices: Real Numbers

Real (Integers)

2D-array

matrix i → a_{ij} = $A_{m\times n}$

m×n: dimension, type, size

A_{ij} → i^{th} row j^{th} column entry

$[a_{ij}]$, a_{ij}, A_{ij}

Operations on Matrices

inputs → output

1) Matrix Addition

$$A_{m\times n} + B_{m\times n} = C_{m\times n}$$

For all i,j $\quad C_{ij} = A_{ij} + B_{ij}$

2) Scalar Multiplication

$c \in \mathbb{R}$, $A \in \mathbb{R}^{m\times n}$ (input) → $B_{m\times n}$ (output)

For all i,j $\quad B_{ij} = c \cdot A_{ij}$

3) Matrix Multiplication

m×n ≠ n×m (m: row, n: column)

Ex

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & -2 & 1 & 0 \\ 3 & 4 & 2 & -1 \end{bmatrix}_{3\times4} \quad B = \begin{bmatrix} -4 & -2 & -1 & 3 \\ 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}_{3\times4}$$

$$A + B = \begin{bmatrix} -3 & 0 & 2 & 7 \\ 6 & 0 & 4 & 4 \\ 8 & 10 & 9 & 7 \end{bmatrix}_{3\times4}$$

Ex

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & -2 & 1 & 0 \\ 3 & 4 & 2 & -1 \end{bmatrix}_{3\times4} \qquad -1/3 \in \mathbb{R}$$

$$\underset{\text{real number}}{-1/3} \cdot \underset{\text{matrix}}{A_{3\times4}} = \begin{bmatrix} -1/3 & -2/3 & -1 & -4/3 \\ -5/3 & 2/3 & -1/3 & 0 \\ -1 & -4/3 & -2/3 & 1/3 \end{bmatrix}_{3\times4}$$

Dot Product : $(x_1, x_2, \dots, x_n) \cdot (y_1, y_2, \dots, y_n) = x_1y_1 + x_2y_2 + \dots + x_ny_n$

Dot Product :

$(x_1, x_2, \dots, x_n) \cdot (y_1, y_2, \dots, y_n) = x_1y_1 + x_2y_2 + \dots + x_ny_n$

ordered n-tupple — dot product — ordered n-tupple

output is a scalar $\in \mathbb{R}$

Ex/ $\underbrace{(1, 2, 3, 4, 5)}_{5\text{-tupple}} \cdot \underbrace{(-2, 3, 1, -4, -7)}_{5\text{-tuple}} = 1.2 + 2.3 + 3.1 + 4.(-4) + 5.(-7)$

$= -2 + 6 + 3 - 16 - 35 = -45 \in \mathbb{R}$ output

Matrix Multiplication

!! → $\underset{\text{left}}{A_{m\times n}} \cdot \underset{\text{right}}{B_{n\times t}} = C_{m\times t}$

$B_{n\times t} \cancel{\cdot} A_{m\times n}$ → multiplication is not possible!

! not every matrix can be multiplied with any other

! The fact that the multiplication AB is possible <u>does not</u> imply BA is also possible.

The order is important

Ex/ $A_{3\times 4} \cdot B_{4\times 13}$ ✓

$A_{3\times 2} \cdot B_{3\times 2}$ ✗ ($2 \neq 3$)

$A_{m\times n} \cdot B_{n\times t} = C_{m\times t}$

$C_{ij} = A_{i^{th}\text{ row}} \cdot B_{j^{th}\text{ column}}$ → real number.

dot product.

A (n-tupple) B

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}_{m\times n} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1t} \\ b_{21} & b_{22} & \dots & \\ \vdots & & & \\ b_{n1} & b_{n2} & \dots & b_{nt} \end{bmatrix}_{n\times t} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{m\times t}$$

(n-tupple)

Ex/ $A = \begin{bmatrix} -1 & 2 & 3 & 4 \\ -2 & 0 & -5 & -3 \end{bmatrix}_{2\times 4}$ $B = \begin{bmatrix} 2 & 3 & -1 \\ -3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & -3 & -4 \end{bmatrix}_{4\times 3}$

mult. is possible ✓

$AB = \begin{bmatrix} -5 & -1 & 4 \\ -9 & -7 & -1 \end{bmatrix}_{2\times 3}$ ✓

1,1 entry = $A_{1^{st}\text{ row}} \cdot B_{1^{st}\text{ column}} = -2 + -6 + 3 + 0 = -5$

1,2 entry = // $\cdot B_{2^{nd}\text{ column}} = -3 + 8 + 6 + -12 = -1$

1,3 entry = // $\cdot B_{3^{rd}\text{ colum}} = 1 + 10 + 9 + -16 = 4$

2,1 entry = $A_{2^{nd}\text{ row}} \cdot B_{1^{st}\text{ column}} = -4 + 0 + -5 + 0 = -9$

2,2 entry = // $\cdot B_{2^{nd}\text{ column}} = -6 + 0 + -10 + 9 = -7$

2,3 entry = // $\cdot B_{3^{rd}\text{ column}} = 2 + 0 + -15 + 12 = -1$

2. For each of the pairs of matrices that follow, determine whether it is possible to multiply the first matrix times the second. If it is possible, perform the multiplication.

(a) $\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

(f) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix}_{2\times 2} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 36 & 10 & 56 \\ 10 & 3 & 16 \end{bmatrix}_{2\times 3}$ ✓