

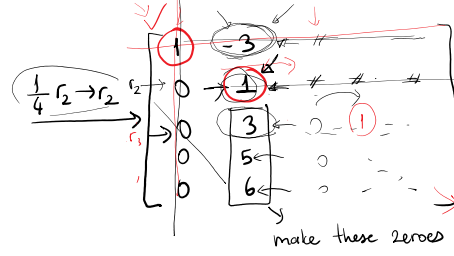
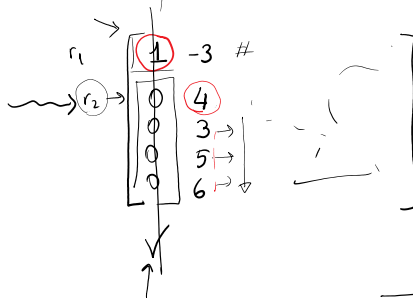
Fixed quizzes available every Saturday $\begin{bmatrix} \downarrow & \downarrow \\ 9 & 9 \end{bmatrix}$ ←
[20min - (25min)] 2-3 question

→ Random quizzes in-class 1-question }

→ 1) $r_i \leftrightarrow r_j$
2) $cr_i \rightarrow r_i$ } → obtain a "leading 1"

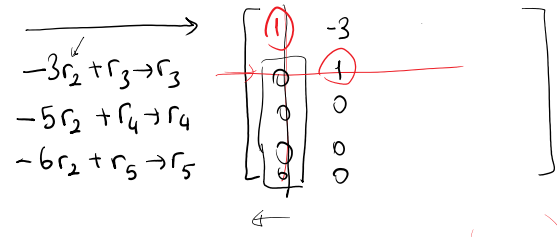
→ 3) $cr_i + r_j \rightarrow r_j$ → making zeros downside → REF (Upside) of a $\begin{bmatrix} \text{REF} \\ \text{REF} \end{bmatrix}$ leading 1.

Ex/ system → augmented matrix



$r_1 + r_3 \rightarrow r_3$ ✓

$-15/8, 93/12$



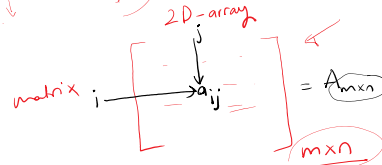
REF

System of LE → Matrices

Matrix Algebra

(world) space of matrices → $\mathbb{R}^{m \times n}$ (m rows, n column) → entries of matrices Real Numbers

Real (Integers)



A_{ij} → ith row jth column entry
 $[a_{ij}], a_{ij}, A_{ij}$

↑ dimension, type, size

Operations on Matrices

Inputs → Output

1) Matrix Addition

$$A_{m \times n} + B_{m \times n} = C_{m \times n}$$

For all i, j $C_{ij} = A_{ij} + B_{ij}$

2) Scalar Multiplication



For all i, j $B_{ij} = c \cdot A_{ij}$

3) Matrix Multiplication

Ex/ $m \times n \neq n \times m$
row column

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & -2 & 1 & 0 \\ 3 & 4 & 2 & -1 \end{bmatrix}_{3 \times 4}$$

$$B = \begin{bmatrix} -4 & -2 & -1 & 3 \\ 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}_{3 \times 4}$$

$$A+B = \begin{bmatrix} -3 & 0 & 2 & 7 \\ 6 & 0 & 4 & 4 \\ 8 & 10 & 9 & 7 \end{bmatrix}_{3 \times 4}$$

Ex/ $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & -2 & 1 & 0 \\ 3 & 4 & 2 & -1 \end{bmatrix}_{3 \times 4}$

$-1/3 \in \mathbb{R}$

$-1/3 \cdot A = \begin{bmatrix} -1/3 & -2/3 & -1 & -4/3 \\ 5/3 & 2/3 & -1/3 & 0 \\ -1 & -4/3 & -2/3 & 1/3 \end{bmatrix}_{3 \times 4}$

Dot Product: $(x_1, x_2, \dots, x_n) \cdot (y_1, y_2, \dots, y_n) = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

Dot Product: $(x_1, x_2, \dots, x_n) \cdot (y_1, y_2, \dots, y_n) = x_1y_1 + x_2y_2 + \dots + x_ny_n$
 ordered n-tuple dot product ordered n-tuple output is a scalar $\in \mathbb{R}$

EX $(1, 2, 3, 4, 5) \cdot (-2, 3, 1, -4, -7) = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 1 + 4 \cdot (-4) + 5 \cdot (-7)$
 $= -2 + 6 + 3 - 16 - 35 = -45 \in \mathbb{R}$
 5-tuple 5-tuple output

Matrix Multiplication

$A_{m \times n} \cdot B_{n \times t} = C_{m \times t}$
 left right

! not every matrix can be multiplied with any other

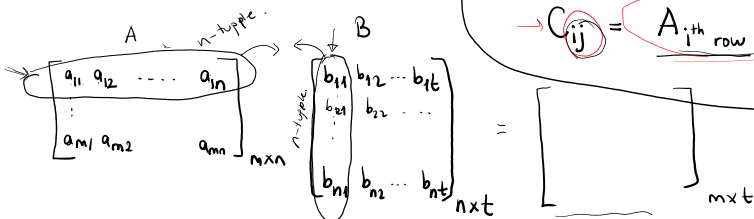
! The fact that the multiplication AB is possible does not imply BA is also possible.

$B_{n \times t} \cdot A_{m \times n} \rightarrow$ multiplication is not possible!

The order is important

EX $A_{3 \times 4} \cdot B_{4 \times 13} \checkmark$
 $A_{3 \times 2} \cdot B_{3 \times 2} \times$

$A_{m \times n} \cdot B_{n \times t} = C_{m \times t}$
 $C_{ij} = A_{i \text{th row}} \cdot B_{j \text{th column}}$ (dot product)
 real number.



EX $A = \begin{bmatrix} -1 & 2 & 3 & 4 \\ -2 & 0 & -5 & -3 \end{bmatrix}_{2 \times 4}$
 $B = \begin{bmatrix} 2 & 3 & -1 \\ -3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & -3 & -4 \end{bmatrix}_{4 \times 3}$
 mult. is possible \checkmark

$AB = \begin{bmatrix} -5 & -1 & 4 \\ -9 & -7 & -1 \end{bmatrix}_{2 \times 3} \checkmark$

- 1,1 entry = $A_{1st \text{ row}} \cdot B_{1st \text{ column}} = -2 + -6 + 3 + 0 = -5$
- 1,2 entry = " $\cdot B_{2nd \text{ column}} = -3 + 8 + 6 + -12 = -1$
- 1,3 entry = " $\cdot B_{3rd \text{ column}} = 1 + 10 + 9 + -16 = 4$
- 2,1 entry = $A_{2nd \text{ row}} \cdot B_{1st \text{ column}} = -4 + 0 + -5 + 0 = -9$
- 2,2 entry = " $\cdot B_{2nd \text{ column}} = -6 + 0 + -10 + 9 = -7$
- 2,3 entry = " $\cdot B_{3rd \text{ column}} = 2 + 0 + -15 + 12 = -1$

2. For each of the pairs of matrices that follow, determine whether it is possible to multiply the first matrix times the second. If it is possible, perform the multiplication.

- (a) $\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$
- (b) $\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$
- (d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$
- (e) $\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$
- (f) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} 36 & 10 & 56 \\ 10 & 3 & 16 \end{bmatrix}_{2 \times 3}$